Section 4.4: Evaluating Logarithms and the Change of Base Formula

Video 1

The ***common logarithm*** $\log_{10} x$ is the logarithm whose base is 10.
It is often written without the base: $\log x$.

All scientific calculators have a built-in function for finding common logarithms.

1) Find the following. Round to 3 decimal places if necessary.

a) $\log 10,000$ b) $\log 321$ c) $\log 0.03$

Video 2

pH of a Solution: $\text{pH} = -\log\left[\text{H}_3\text{O}^+\right]$
where $\left[\text{H}_3\text{O}^+\right]$ is the hydronium ion concentration in moles/liter.

2) Find the pH of a solution with $\left[\text{H}_3\text{O}^+\right] = 3.2x10^{-5}$.

3) Find the hydronium ion concentration in moles/liter for a substance whose pH is 9.6.

Video 3

Decibel Rating of a Sound: $d = 10\log\frac{I}{I_0}$, where I_o is the intensity of the threshold sound and *I* is the intensity of the sound.

4) Find the decibel rating of a sound whose intensity is $75,000I_0$.

5) Find the intensity of the sound of a vacuum cleaner whose decibel rating is 85.

Video 4

The natural logarithm, $\ln x$, is the logarithm whose base is the natural base $e \approx 2.71828...$

6) Find the following. Round to 3 decimal places if necessary.

a) $\ln e^7$

b) $\ln 2000$

c) $\ln 0.01$

Video 5

The age of a rock, *t*, can be determined by measuring its amounts of argon-40 (*A*) and potassium-40 (*K*), and using this formula: $t = \left(1.26 \times 10^9\right) \dfrac{\ln\left(1 + 8.33\left(\dfrac{A}{K}\right)\right)}{\ln 2}$.

7) Find the age of a rock in which $A = 4.5K$. Round to the nearest hundredth of a billion years.

8) Find the age of a rock for which $\dfrac{A}{K} = 0.8$. Round to the nearest hundredth of a billion years.

Video 6

Change of Base formula

For any positive numbers *a*, *b*, and *x* $\left(a \neq 1, b \neq 1\right)$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

This allows us to rewrite any logarithm in terms of natural logarithms or common logarithms.

Proof:

9) Use the Change of Base formula to approximate the following to 4 decimal places.

a) $\log_2 32$

b) $\log_6 50$

c) $\log_9 0.5$